SOLID STATE PHYSICS 1

- January 21st, 2008 -

Do not forget to write your full name and student number on <u>each</u> sheet.
Please use <u>separate</u> sheets for each of the four problems.

Planck's constant	$\hbar$	1.055×10^{-34} Js
Planck's constant	h	6.626×10^{-27} erg s
Boltzmann's constant	k_B	1.38×10^{-23} J K^{-1}
Permittivity of free space	ε_0	8.85×10^{-12} F m^{-1}
Permeability of free space	μ_0	$4\pi \times 10^{-7}$ H m^{-1}
speed of light	c	3.0×10^{8} m s^{-1}
elementary charge	e	1.60×10^{-19} C
mass of the electron	m	9.11×10^{-31} kg
Bohr magneton	μ_B	9.27x10-24 J T^{-1}
Note also: 1 eV = 1.6019×10^{-12} erg		

I. The following questions should be answered very briefly (1-2 sentences);

(a) By which techniques (name one for each property) can one experimentally determine the following:
- lattice constant of diamond
- phonon dispersion curves of copper
- valence band density of silver
- the band gap of gallium arsenide

(b) What relationship exists between the thermal conductivity κ and the electrical conductivity σ of a metal at not too low temperature?

(c) In what kind of crystals do you expect to find the following? (In some cases, the answer may be "all crystals")
i) Fermi surface.
ii) Acceptors and/or donors.
iii) Acoustic phonon branches.
iv) Thermal conductivity dominated by phonon (not electronic) contribution.

(d) A system of electron spins (j=s=1/2, g=2) is placed is a magnetic field B = 5T along z. What are the possible energy levels for one spin in such field, and what is the energy splitting between them? (Please calculate!)

(e) What important property of an electron gas is determined by measuring the Hall coefficient? What is set and what is measured in such an experiment?

(f) Assume Bi_2Te_3 crystallizes with one molecule (of five atoms) in a unit cell (not quite true in nature). Knowing that Bi is in column V, and Te in column VI of the periodic table arguments based on band theory to tell if this material is a metal or an insulator and give the number of filled and half-filled bands.

(g) Apply Hund's rule to find the angular momentum quantum number, the spin quantum number and the total angular + spin momentum quantum number for Sm^{2+} in the configuration $4f^6$

(h) What is a semiconductor quantum dot?

(i) How do you recognize a piezoelectric crystal? Name one practical application for piezoelectric crystals.

(j) Name two properties of a crystal which change when the crystal dimensions are reduced to the nanometer scale.

II. For the fcc lattice with lattice constant a, find:

(a) the three primitive translation vectors [1.5 points]

(b) the three primitive reciprocal lattice vectors [1.5 points]

(c) the number of lattice points per unit volume [1 point]

(d) the number of nearest neighbours [1 points]

(e) the packing fraction (sphere volume divided by primitive cell volume when spheres are packed together in contact) [1 point]

(f) why for an inert gas this is the energetically preferred structure and not bcc. [3 points] Remember that the cohesive energy of an inert gas whose N atoms interact via the Lennard-Jones potential:

$$U_{coh}(r) = \frac{N}{2}\sum_{ij} U_{ij},$$

$$U_{ij} = 4\varepsilon\left[\left(\frac{\sigma}{r_{ij}}\right)^{12} - \left(\frac{\sigma}{r_{ij}}\right)^{6}\right]$$

and that the lattice sums $A_6 = \sum_j' p_{ij}^{-6}$ and $A_{12} = \sum_j' p_{ij}^{-12}$ where $p_{ij}r$ is the distance between reference atom i and any other atom j are given in the table below for bcc and fcc crystal structures.

	bcc	*fcc*
A_6	12.2533	14.4539
A_{12}	9.1142	12.1319

III. The energy near a valence band edge is given by $E_k = -1 \times 10^{-26} k^2$ ergs. An electron is removed from the orbital $k = 1 \times 10^7 k_x$ cm^{-1}. The band is otherwise full.

(a) Give the sign and the magnitude of the effective mass of the hole. [2 points]

(b) Does a negative effective mass of an electron at the centre of the Brillouin zone mean an upward or a downward curvature of the band? [1 point]

(c) What is the direction and the magnitude of the wave vector of the hole? [1 point]

(d) What is the crystal momentum of the hole? [1 point]

(e) What is the velocity of the hole? [2 points]

(f) What is the energy of the hole referred to the valence band edge? [1 point]

(g) What is the electric current carried by the hole? [2 points]

IV. A crystalline body in a state of thermally exited elastic vibration may be treated as a system of N distinguishable independent quantum harmonic oscillators of the same angular frequency ω (Einstein's model).

(a) Give the expression for the probability that a given level n is occupied. [1 point]

(b) Compute the average energy of the system at high and low temperatures. [2 points]

(c) Find the molar specific heat in those limiting cases. [2 points]

(d) Discuss the validity of the model in those cases. [2points]

(e) Do you know another model which gives better results either at high or at low temperature? How does it differ? [2 points]

(f) Phonons are not only conducting heat through the crystal, what else do they conduct? [1 point]
